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CHARACTERISTIC PROPERTIES OF THE SYSTEM OF EQUATIONS OF A SHEAR FLOW WITH NONMONOTONIC VELOCITY PROFILE

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1. Formulation of the Problem. We consider the initial boundary value problem with a free boundary

$$\begin{gathered} \rho(U_T + UU_X + VU_Y) + p_X = 0, \quad 0 \leqslant Y \leqslant H(X,T), \\ U_X + V_Y = 0, \quad p_Y = -\rho g, \quad H_T + \Big(\int\limits_0^H U\,dY\Big)_X = 0, \\ p(X, H(X,T), T) = 0, \quad V(X,0,T) = 0, \\ U(X,Y,0) = U_0(X,Y), \quad H(X,0) = H_0(X), \end{gathered} \tag{1.1}$$

which describes, in the long-wave theory approximation, the plane-parallel vortex flow of a layer of a homogeneous liquid at a depth $H = H(X,T)$ above an even bottom $Y = 0$ in a gravity field. Here U, V are components of the vector of the liquid velocity, p is the pressure, ρ is the density ($\rho = \text{const}$), g is the acceleration of gravity, and $U_0(X,Y)$, $H_0(X)$ are prescribed functions.

It was shown in [1] that the problem (1.1) is reduced to the Cauchy problem for the system of integro-differential equations

$$\begin{gathered} u_t + uu_x + g\int\limits_0^1 h_x\,d\nu = 0, \quad h_t + (uh)_x = 0, \quad 0 \leqslant \lambda \leqslant 1, \\ u(x,0,\lambda) = U_0(x, \lambda H_0(x)), \quad h(x,0,\lambda) = H_0(x), \end{gathered} \tag{1.2}$$

where $u(x,t,\lambda) = U(x, \Phi(x,t,\lambda), t)$, $h(x,t,\lambda) = \Phi_\lambda(x,t,\lambda)$ and $\Phi(x,t,\lambda)$ results from the solution of the problem

$$\Phi_t + \Big(\int\limits_0^\Phi U(x,Y,t)\,dY\Big)_x = 0, \quad \Phi(x,0,\lambda) = \lambda H_0(x).$$

The surfaces $\lambda = \text{const}$ so defined are contact surfaces; $\lambda = 0$ corresponds to the bottom, $\lambda = 1$ corresponds to a free surface. When $U_Y \equiv 0$ (in the long-wave approximation this corresponds to a vortex-free flow) Eqs. (1.2) become the well-known equations of shallow water theory which are of the hyperbolic type. The question of the type of Eqs. (1.2) arises when considering vortex flows with $U_Y \not\equiv 0$.

In [1, 2] hyperbolicity is defined for a system with operator coefficients, and the conditions of hyperbolicity for Eqs. (1.2) are determined for a monotonic velocity profile ($U_Y \neq 0$). Here we obtain the conditions of hyperbolicity for Eqs. (1.2) for a nonmonotonic velocity profile under the assumption that U_Y vanishes at a single point $Y_*(X,T)$, $0 < Y_*(X,T) < H(X,T)$. In this case $U_{YY}(Y_*) \neq 0$.

It should be noted that the conditions of hyperbolicity play an important role in the analysis of shear flow stability because their violation results in ill-posedness of the Cauchy problem for the system of equations of fluid motion.

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According to [1], the determination of the type of Eqs. (1.2) is reduced to searching for eigenfunctionals φ and eigenvalues k satisfying the equation

$$(\varphi, A\mathbf{f}) = k(\varphi, \mathbf{f}). \tag{1.3}$$

The operator A is defined by the equality

$$A(f_1, f_2)^t(\lambda) = \left(u(\lambda)f_1(\lambda) + g\int_0^1 f_2(\nu)\,d\nu, \quad h(\lambda)f_1(\lambda) + u(\lambda)f_2(\lambda)\right)^t.$$

Here $(\varphi, \mathbf{f})$ denotes the action of the functional φ on the test function $\mathbf{f}$; $\mathbf{f} = (f_1, f_2)^t$ is a sufficiently smooth function; the index t denotes transposition.

Since f_1 and f_2 are independent, Eq. (1.3) is equivalent to the following:

$$(\varphi_1, (u-k)f_1) + (\varphi_2, hf_1) = 0,$$

$$g\int_0^1 f_2\,d\nu(\varphi_1, 1) + (\varphi_2, (u-k)f_2) = 0.$$

If the functional φ_1 is known, then the action of the functional φ_2 is given by the formula

$$(\varphi_2, f) = -(\varphi_1, (u-k)h^{-1}f).$$

To find the functional φ_1 and eigenvalues k, we solve the equation

$$-(\varphi_1, (u-k)^2h^{-1}f_2) + g\int_0^1 f_2\,d\nu(\varphi_1, 1) = 0.$$

For brevity, the arguments x, t are omitted.

For Eqs. (1.2) to be hyperbolic, all the eigenvalues k^α which satisfy Eq. (1.3) are required to be real and the set of corresponding eigenfunctionals $\{\varphi^\alpha\}$ to be complete, i.e., the equalities $(\varphi^\alpha, \mathbf{f}) = 0$ should involve the equality $\mathbf{f} \equiv 0$. When the conditions of hyperbolicity are fulfilled, Eqs. (1.2) can be transformed into the equivalent characteristic form: $(\varphi^\alpha, \mathbf{u}_t + k^\alpha \mathbf{u}_x) = 0$, where $\mathbf{u} = (u, h)^t$, k^α are the eigenvalues corresponding to φ^α.

2. The Derivation of Eigenfunctions.

Lemma 2.1. *The eigenvalues k_i of a discrete spectrum are specified by the equation*

$$g\int_0^1 h(\nu)(u(\nu) - k_i)^{-2}\,d\nu = 1. \tag{2.1}$$

Proof. This lemma is proved by direct verification that the eigenfunctionals $\varphi^i = (\varphi_1^i, \varphi_2^i)$ satisfy Eq. (1.3) with k_i the roots of Eqs. (2.1). The action of φ^i on an arbitrary smooth function $\mathbf{f}(\mathbf{f} = (f_1, f_2)^t)$ is given by the formulas:

$$(\varphi_1^i, f_1) = \int_0^1 f_1(\nu)h(\nu)(u(\nu) - k_i)^{-2}\,d\nu, \quad i = 1, 2,$$

$$(\varphi_2^i, f_2) = -\int_0^1 f_2(\nu)(u(\nu) - k_i)^{-1}\,d\nu, \quad i = 1, 2.$$

Equation (2.1) always has two real roots outside of the segment $[\min_\lambda u(x,t,\lambda), \max_\lambda u(x,t,\lambda)]$; generally speaking, it may also have complex roots.

The velocity profile that satisfies the conditions

$$u_\lambda > 0 \text{ for } 0 < \lambda < \lambda_1(x,t), \quad u_\lambda < 0 \text{ for } \lambda_1(x,t) < \lambda < 1,$$
$$u_{\lambda\lambda}(\lambda_1(x,t)) \neq 0, \quad u(0) < u(1). \tag{2.2}$$

is considered.

Let $\lambda_2(x,t)$ be a point in $[0,1]$, where the equality $u(\lambda_2) = u(1)$ is achieved. For each point λ from $[\lambda_2, \lambda_1]$ we define the function $\lambda_s = \lambda_s(\lambda, x, t)$ ($\lambda_s \geqslant \lambda_1$) given by the equality $u(\lambda, x, t) = u(\lambda_s(\lambda, x, t), x, t)$. In what follows the arguments x, t in the notation of the functions $\lambda(x,t)$, $\lambda_s(x,t)$ are omitted for brevity. For any smooth function ψ there is a 3rd-degree polynomial in ν $Q(\nu, \lambda, \lambda_s, \psi)$ meeting the conditions

$$Q(\lambda, \lambda, \lambda_s, \psi) = \psi(\lambda), \qquad Q(\lambda_s, \lambda, \lambda_s, \psi) = \psi(\lambda_s),$$
$$Q_\nu(\lambda, \lambda, \lambda_s, \psi) = \psi_\nu(\lambda), \quad Q_\nu(\lambda_s, \lambda, \lambda_s, \psi) = \psi_\nu(\lambda_s).$$

This polynomial can be represented as

$$Q(\nu, \lambda, \lambda_s, \psi) = \frac{1}{2}(\psi(\lambda) + \psi(\lambda_s)) - \frac{1}{8}(\psi_\nu(\lambda) - \psi_\nu(\lambda_s))(\lambda - \lambda_s)$$
$$+(-\frac{1}{4}(\psi_\nu(\lambda) + \psi_\nu(\lambda_s)) + \frac{3}{2}(\psi(\lambda) - \psi(\lambda_s))(\lambda - \lambda_s)^{-1})(\nu - \frac{1}{2}(\lambda + \lambda_s))$$
$$+\frac{1}{2}(\psi_\nu(\lambda) - \psi_\nu(\lambda_s))(\lambda - \lambda_s)^{-1}(\nu - \frac{1}{2}(\lambda + \lambda_s))^2$$
$$+((\psi_\nu(\lambda) + \psi_\nu(\lambda_s))(\lambda - \lambda_s)^{-2} - 2(\psi(\lambda) - \psi(\lambda_s))(\lambda - \lambda_s)^{-3})(\nu - \frac{1}{2}(\lambda + \lambda_s))^3.$$

The polynomial $Q(\nu, \lambda, \lambda_s, \psi)$ is constructed in such a way that the difference between the values of $Q(\nu, \lambda, \lambda_s, \psi)$ and $\psi(\nu)$ in the neighborhood of the point λ_1 (where $\lambda \to \lambda_s$) can be estimated as $(\psi(\nu) - Q(\nu, \lambda, \lambda_s, \psi)) = O((\nu - \lambda_1)^4)$.

We introduce one more 3rd-degree polynomial in ν $Q_1(\nu, \lambda, \lambda_s, \psi)$ given by the formula

$$Q_1(\nu, \lambda, \lambda_s, \psi) = \frac{1}{8}(\omega(\lambda)\psi(\lambda) - \omega(\lambda_s)\psi(\lambda_s))(\lambda - \lambda_s)$$
$$+\frac{1}{4}(\omega(\lambda)\psi(\lambda) + \omega(\lambda_s)\psi(\lambda_s))(\nu - \frac{1}{2}(\lambda + \lambda_s))$$
$$-\frac{1}{2}(\omega(\lambda)\psi(\lambda) - \omega(\lambda_s)\psi(\lambda_s))(\lambda - \lambda_s)^{-1}(\nu - \frac{1}{2}(\lambda + \lambda_s))^2$$
$$-(\omega(\lambda)\psi(\lambda) + \omega(\lambda_s)\psi(\lambda_s))(\lambda - \lambda_s)^{-2}(\nu - \frac{1}{2}(\lambda + \lambda_s))^3,$$

where $\omega(\lambda) = u_\nu(\lambda)/h(\lambda)$.

Consider the functionals $\delta(\lambda)$, $\delta'(\lambda)$ [4], $P_{10}(\lambda)$ ($\lambda \in (0, \lambda_2)$), $P_{11}(\lambda)$ ($\lambda \in (0, \lambda_2)$), $P_0(\lambda)$ ($\lambda \in (\lambda_2, 1)$), $P_1(\lambda)$ ($\lambda \in (\lambda_2, 1)$) acting on a smooth test function ψ according to the following rules:

$$(\delta(\lambda), \psi) = \psi(\lambda), \quad (\delta'(\lambda), \psi) = -\psi_\nu(\lambda),$$

$$(P_{10}(\lambda), \psi) = \int_0^1 h(\nu) \frac{\psi(\nu) - \psi(\lambda)}{(u(\nu) - u(\lambda))^2}\, d\nu, \quad \lambda \in (0, \lambda_2),$$

$$(P_{11}(\lambda), \psi) = \int_0^1 \frac{\psi(\nu)\, d\nu}{u(\nu) - u(\lambda)}, \quad \lambda \in (0, \lambda_2),$$

$$(P_0(\lambda), \psi) = \int_0^{\lambda_2} h(\nu) \frac{\psi(\nu) - \psi(\lambda)}{(u(\nu) - u(\lambda))^2}\, d\nu + \int_{\lambda_2}^1 h(\nu) \frac{\psi(\nu) - Q(\nu, \lambda, \lambda_s, \psi)}{(u(\nu) - u(\lambda))^2}\, d\nu, \qquad \lambda \in (\lambda_2, 1),$$

$$(P_1(\lambda),\psi) = \int\limits_0^{\lambda_2} \frac{\psi(\nu)\,d\nu}{u(\nu)-u(\lambda)} + \int\limits_{\lambda_2}^{1} \frac{\psi(\nu)(u(\nu)-u(\lambda)) - h(\nu)Q_1(\nu,\lambda,\lambda_s,\psi)}{(u(\nu)-u(\lambda))^2}\,d\nu, \qquad \lambda \in (\lambda_2, 1).$$

Lemma 2.2. *For every eigenvalue $k^\lambda = u(x,t,\lambda)$, $(\lambda \in (0,\lambda_2))$ there are two eigenfunctionals ($\varphi^{11\lambda}$ and $\varphi^{21\lambda}$) given by the formulas*

$$\varphi^{11\lambda} = \Big(\delta'(\lambda), \omega(\lambda)\delta(\lambda)\Big), \quad \varphi^{21\lambda} = \Big(gP_{10}(\lambda) + \delta(\lambda),\ -gP_{11}(\lambda)\Big).$$

Lemma 2.3. *For every eigenvalue $k^\lambda = u(x,t,\lambda)$ $(\lambda \in (\lambda_2, 1))$ there are four eigenfunctionals ($\varphi^{1\lambda}$, $\varphi^{2\lambda}$, $\varphi^{3\lambda}$, $\varphi^{4\lambda}$) given by the formulas:*

$$\varphi^{1\lambda} = \Big((\delta'(\lambda) + \delta'(\lambda_s)) + 6(\delta(\lambda) - \delta(\lambda_s))(\lambda - \lambda_s)^{-1}; (\omega(\lambda)\delta(\lambda) + \omega(\lambda_s)\delta(\lambda_s))\Big),$$

$$\varphi^{2\lambda} = \Big((\delta'(\lambda) - \delta'(\lambda_s))(\lambda - \lambda_s)^{-1}, (\omega(\lambda)\delta(\lambda) - \omega(\lambda_s)\delta(\lambda_s))(\lambda - \lambda_s)^{-1}\Big),$$

$$\varphi^{3\lambda} = \Big((\delta'(\lambda) + \delta'(\lambda_s))(\lambda - \lambda_s)^{-2} + 2(\delta(\lambda) - \delta(\lambda_s))(\lambda - \lambda_s)^{-3},\ (\omega(\lambda)\delta(\lambda) + \omega(\lambda_s)\delta(\lambda_s))(\lambda - \lambda_s)^{-2}\Big),$$

$$\varphi^{4\lambda} = \Big(gP_0(\lambda) + \delta(\lambda), -gP_1(\lambda)\Big).$$

Proof is obtained after straightforward substitution of the above eigenfunctionals into Eq. (1.3).

3. Conditions of Hyperbolicity for Eqs. (1.2). Let us verify that the constructed set of eigenfunctionals is complete. We obtain a condition which provides an equality $\mathbf{f} = (f_1, f_2)^t = 0$ as a consequence of the relations:

$$\begin{gathered}(\varphi^{11\lambda}, \mathbf{f}) = 0, \quad (\varphi^{21\lambda}, \mathbf{f}) = 0, \quad (\varphi^{1\lambda}, \mathbf{f}) = 0, \quad (\varphi^{2\lambda}, \mathbf{f}) = 0, \\ (\varphi^{3\lambda}, \mathbf{f}) = 0, \quad (\varphi^{4\lambda}, \mathbf{f}) = 0, \quad (\varphi^{1}, \mathbf{f}) = 0, \quad (\varphi^{2}, \mathbf{f}) = 0.\end{gathered} \tag{3.1}$$

For $\lambda \in (0, \lambda_2)$ it results from (3.1) that $f_2 = \omega^{-1}(f_1)_\nu$. For $\lambda \in (\lambda_2, 1)$ it follows from (3.1) that $f_1(\lambda) = f_1(\lambda_s)$, $f_2(\lambda) = \omega^{-1}(\lambda)(f_1)_\nu(\lambda)$, $f_2(\lambda_s) = \omega^{-1}(\lambda_s)(f_1)_\nu(\lambda_s)$. Using these equalities we obtain the integral equations for determination of the function f_1:

$$f_1(\lambda) - g\int\limits_0^1 \frac{1}{\omega}\frac{\partial}{\partial\nu}\Big(\frac{f_1(\nu) - f_1(\lambda)}{u(\nu) - u(\lambda)}\Big)\,d\nu = 0; \tag{3.2}$$

$$\int\limits_0^1 \frac{1}{\omega}\frac{\partial}{\partial\nu}\Big(\frac{f_1(\nu)}{u(\nu) - k_i}\Big)\,d\nu = 0 \quad (i = 1,\ 2). \tag{3.3}$$

It is easy to verify that Eq. (3.2) is fulfilled if f_1 is replaced by the function $f_{11} = \alpha_1(u - k_1)^{-1} + \alpha_2(u - k_2)^{-1}$, where α_1, α_2 are arbitrary values independent of λ.

We search for a general solution to Eq. (3.2) in the form $f_1 = f_{10} + f_{11}$, where f_{10} satisfies the conditions $f_{10}(0) = f_{10}(\lambda_1)$, $f_{10}(1) = f_{10}(\lambda_1)$. These conditions can be fulfilled by the proper choice of α_1, α_2. The function f_{10} has the symmetry property $f_{10}(\lambda) = f_{10}(\lambda_s)$ that follows from the equalities (3.1) .

Integrating by parts and changing variables, we transform Eq. (3.2) into

$$\psi(u)\Big[(u - u_*) + g(u_1 - u_*)(u_1 - u)^{-1}\omega_1^{-1} - g(u_0 - u_*)(u_0 - u)^{-1}\omega_0^{-1} - g\int\limits_{u_0}^{u_*} \frac{\rho(u')\,du'}{u' - u}\Big] + g\int\limits_{u_0}^{u_*} \frac{\rho(u')\psi(u')\,du'}{u' - u} = -f_{10*} \tag{3.4}$$

($\psi(u) = (f_{10}(u) - f_{10*})(u - u_*)^{-1}$). Note that $\psi_0 = \psi_1 = 0$. Here u', u, ω_s are abbreviations for $u(x,t,\nu)$,

$u(x,t,\lambda)$, $\omega(x,t,\lambda_s)$; indices 0, 1, $*$ correspond to the values of the functions when $\lambda = 0$, $\lambda = 1$, $\lambda = \lambda_1$; $\omega(u)$, $\omega_s(u)$, $\psi(u)$, $f_{10}(u)$ are the dependences of the functions ω, ω_s, ψ, f_{10} on $u(\lambda)$.

The function $\rho(u)$, discontinuous at the point u_1, is given by the formulas

$$\rho(u) = (u - u_*)\frac{\partial}{\partial u}\Big(\frac{1}{\omega(u)}\Big) \quad \text{for } u \in (u_0, u_1),$$

$$\rho(u) = (u - u_*)\Big(\frac{\partial}{\partial u}\Big(\frac{1}{\omega(u)}\Big) - \frac{\partial}{\partial u}\Big(\frac{1}{\omega_s(u)}\Big)\Big) \quad \text{for } u \in (u_1, u_*).$$

The function $\rho(u)$ has a peculiarity $\rho = O(|u-u_*|)^{-1/2}$ at the point u_*. Indeed, due to the assumptions (2.2), $(u-u_*) = O((\lambda-\lambda_1)^2)$ in the neighborhood of the point $\lambda = \lambda_1$, and then $|\omega| = |u_\lambda h^{-1}| = O(|\lambda-\lambda_1|) = O(|u-u_*|^{1/2})$. If we introduce the analytical functions

$$\chi(z) \doteq (z - u_*)\Big(1 - g\int_0^1 \frac{h\,d\nu}{(u-z)^2}\Big), \quad F(z) = \int_{u_0}^{u_*} \frac{\rho(u)\psi(u)\,du}{u-z},$$

then the integral equation (3.4) is reduced to the following Riemann problem on the plane of a complex variable z with a cut along the segment $[u_0, u_*]$:

$$F^+(u) = \frac{\chi^+}{\chi^-}F^-(u) + \frac{f_{10*}}{2\pi i g}\Big(\frac{\chi^+}{\chi^-} - 1\Big), \quad u \in [u_0, u_*]. \tag{3.5}$$

Here the $+$ and $-$ signs relate to the limiting values of the functions as $z \to u$ from the upper and lower half-planes. We search for the solution of the problem in the class of functions vanishing at infinity and unbounded at the point u_*.

We extend the boundary condition to the real axis. The function $G = \chi^+/\chi^-$ is assumed to be equal to unity at the segments $]-\infty, u(0)]$, $]u(\lambda_1), +\infty[$. Due to the general theory, the question of the unique solvability of the problem (3.5) is reduced to the determination of its index [3]. The Riemann problem considered here has coefficients that are discontinuous at the point u_* ($G(u_* - 0) = -1$, $G(u_* + 0) = 1$). The index of the Riemann problem in the specified class of solutions æ is equal to -1, and the Riemann problem has a unique solution if $\chi^\pm \neq 0$ when $z \in (u_0, u_*)$ and the equality

$$\frac{1}{\pi}\Delta \arg \frac{\chi^+}{\chi^-} = -3 \tag{3.6}$$

is true, with Δ an increment at the segment (u_0, u_*). Since $\chi(z)$ has poles at the points u_0, u_1 and zeroes at the points k_1, k_2, the canonical solution of the Riemann problem satisfying the boundary condition and having zeroth order everywhere in the finite part of a plane and $(-$æ$)$ order at infinity is of the form

$$X(z) = \frac{(z-u_0)(z-u_1)}{(z-k_1)(z-k_2)}\chi(z).$$

When æ $= -1$, the problem (3.5) has a unique solution if the following condition holds:

$$f_{10*}\int_{u_*}^{u_0} \frac{(\chi^+/\chi^- - 1)(u-k_1)(u-k_2)\,du}{(u-u_0)(u-u_1)\chi^+} = 0. \tag{3.7}$$

Let us show that the factor of f_{10*} is not equal to zero. In the class of functions bounded at infinity problem (3.5) is undoubtedly uniquely solvable (since its index æ $= 0$ is equal to zero). It follows from Eq. (3.5) that the function $F(z) = -f_{10*}/(2\pi i g)$ is the solution to the Riemann problem in the class of functions bounded at infinity. Then the factor of f_{10*} in Eq. (3.7) is not equal to zero. Indeed, if the indicated factor were equal to zero, the Riemann problem (3.5) would also have a solution in the class of functions vanishing at infinity, but it would contradict the unique solvability of the problem in the class of functions bounded at infinity. Hence, it follows from (3.7) that $f_{10*} = 0$. Since the homogeneous Riemann problem has only a trivial solution (æ $= -1$) in the class of functions vanishing at infinity, it follows that $\psi = 0$, and then also $f_{10} = 0$.

Thus, it is found that

$$f_1 = \alpha_1(u - k_1)^{-1} + \alpha_2(u - k_2)^{-1}.$$

Substituting f_1 into Eqs. (3.3) yields

$$\alpha_1 \int_0^1 h(u' - k_1)^{-3}\, d\nu = 0, \quad \alpha_2 \int_0^1 h(u' - k_2)^{-3}\, d\nu = 0. \tag{3.8}$$

Here we use the relationship

$$\int_0^1 h(u' - k_1)^{-2}(u' - k_2)^{-1}\, d\nu + \int_0^1 h(u' - k_1)^{-1}(u' - k_2)^{-2}\, d\nu = 0.$$

It follows from (3.8) that $\alpha_1 = \alpha_2 = 0$, and then $f_1 = 0$. As a result we have proved

Theorem 3.1. *The system of equations* (1.2) *is hyperbolic for the nonmonotonic velocity profile, meeting the conditions* $\chi^{\pm} \neq 0$, (3.6), *and* (2.2).

Note that for the validity of the statement it is sufficient, for solving Eqs. (1.2) $\mathbf{u} = (u, h)^t$, to have the following smoothness:

$$u, u_t, u_x \in C^{2+\alpha}[0,1], \quad h, h_t, h_x \in C^{1+\alpha}[0,1] \quad (0 < \alpha < 1).$$

Applying the above eigenfunctionals, it is possible to reduce the system of equations (1.2) to the relationships on characteristics.

Hyperbolicity of the equations allows us to describe the propagation of disturbances in a fluid and to find the influence regions of initial and boundary data and the regions of existence and uniqueness of solution of initial boundary value problems.

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